Scientific Computing in Computer Science
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Full Waveform Inversion

using Fourier Neural Operators and an adversarial regularization network

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Abstract

We propose a workflow to solve Full Waveform Inversion problems with regularization to detect defects in concrete structures. An **actuator** creates an ultrasound wave that travels through the structure and changes it speed when hitting defects inside it. Then, a **sensor array** on the outside of the structure observes the waves propagating back. From these measurements, the internal defects in the structure can be inferred. Fourier Neural Operators are used to map actuator data to the wave inside the structure at certain time points. Inspired by work in computer vision for computer tomography in medicine, we also train a regularizer term for the outer inverse problem, using an Adversarial Regularization Network.

Algorithm

Full waveform inversion

Given measurement $U_s(x_s, y_s, t) \quad \forall(x_s, y_s) \in \Omega_s, t \in T$
Guess initial defect Ω_g
Define $r = \infty, n_{it} = 0, M_{it}, \epsilon$
while $r > \epsilon$ or $n_{it} < M_{it}$ **do**
    Evaluate wave $U_e(x_s, y_s, t)$ over Ω_g
    Evaluate regularization term $\Phi_\theta(\Omega_g)$
    Evaluate loss $L = ||U_e - U_s||_2^2 + \Phi_\theta(\Omega_g)$
    Update $\Omega_g \leftarrow \Omega_g - \alpha \nabla_{\Omega_g} L$
end while
return Ω_g

Computational Domain

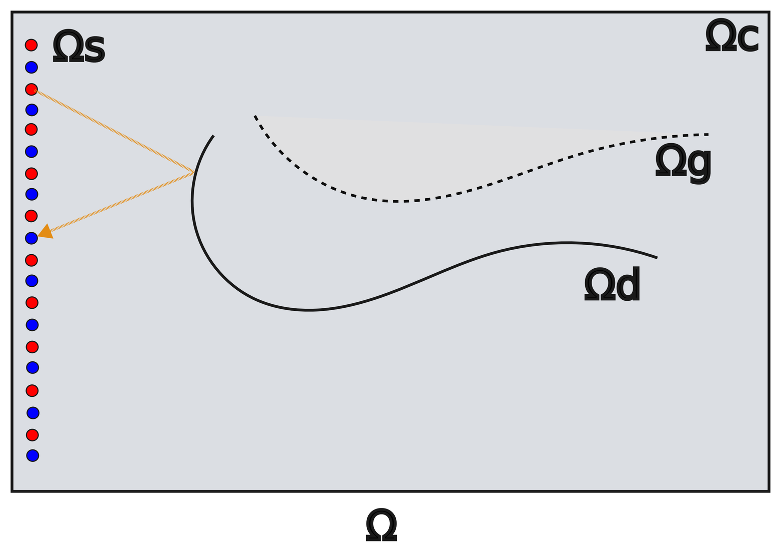


Figure 1 Computational Setup

Forward Problem - U_e

The propagation of waves in Ω; enclosed by boundary $\delta\Omega$; over a time T, is modelled using the acoustic wave equation with homogenous Neumann boundary conditions.

$$\partial_{tt} u(x, y, t) = c_\omega^2 \left(\nabla_{xx} u(x, y, t) + \nabla_{yy} u(x, y, t)\right) + s(x, y) \quad (1)$$
$$\partial_x u(x, y, t) = 0 \quad \forall (x, y) \in \delta\Omega \cup \Omega_g \quad (2)$$
$$\partial_y u(x, y, t) = 0 \quad \forall (x, y) \in \delta\Omega \cup \Omega_g \quad (3)$$

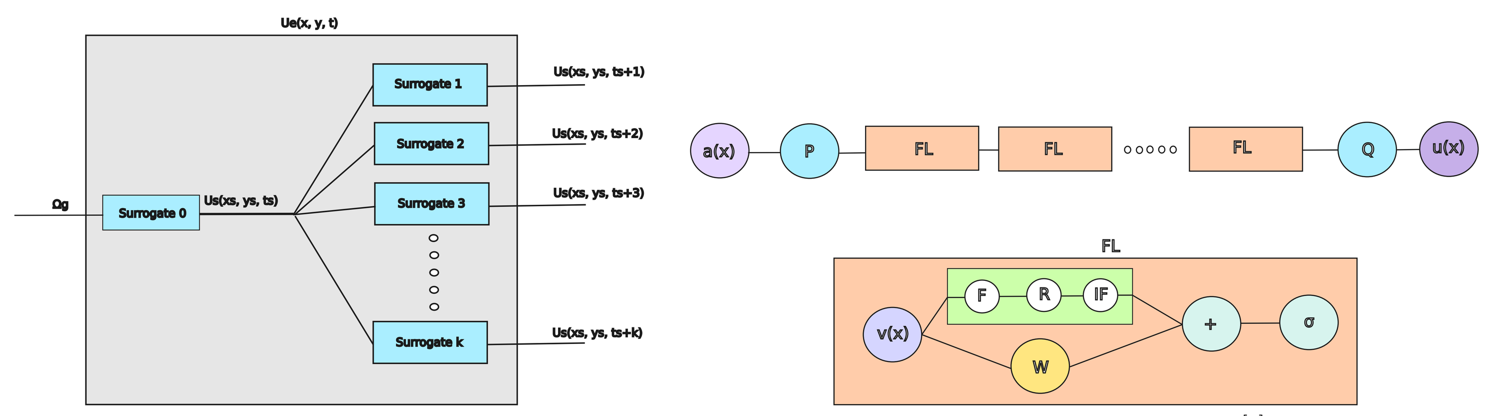
where

$$s(x, y, t) = \begin{cases} A_0 \exp^{\left(\frac{x-\bar{x}}{\sigma_x}\right)^2 + \left(\frac{y-\bar{y}}{\sigma_y}\right)^2} & \forall t \in [0, t_s) \\ 0 & \forall t \in [t_s, t_e] \end{cases}$$

and

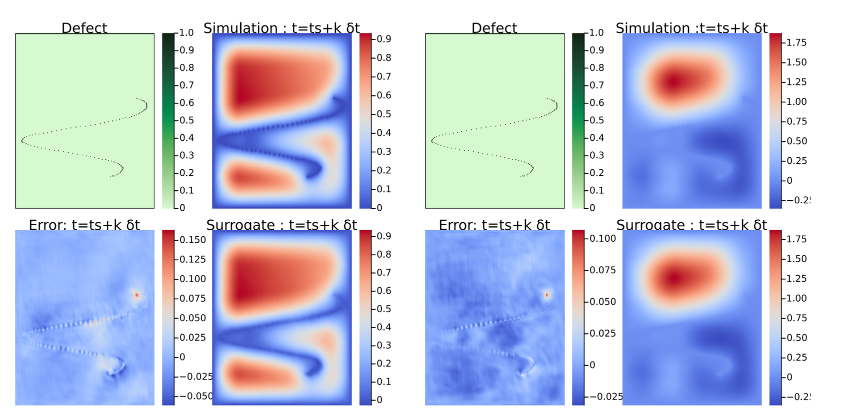
$$c_\omega(x, y) = \begin{cases} c_c & \forall (x, y) \in \Omega_c \\ 0 & \forall (x, y) \in \Omega_g \end{cases}$$

where $A_0, \bar{x}, \bar{y}, \sigma_x, \sigma_y$ are constants chosen for a suitable initial condition and c_c is the velocity of sound in concrete.



(a) Surrogate model **(b)** Fourier Neural Operator[1]

Figure 2 Layouts for Surrogate Model and a single layer of FNO.



(a) $k = 1$ **(b)** $k = 39$

Figure 3 Numerical Simulation at $t = t_s + \delta t$ and $t = t_s + k\delta t$ vs Predictions from Surrogate.

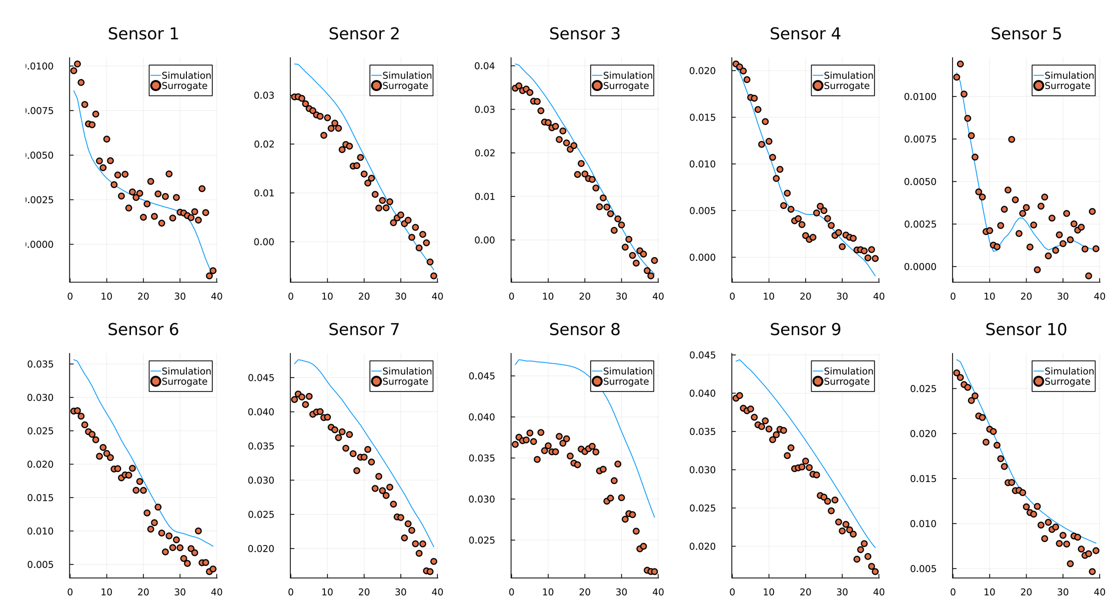


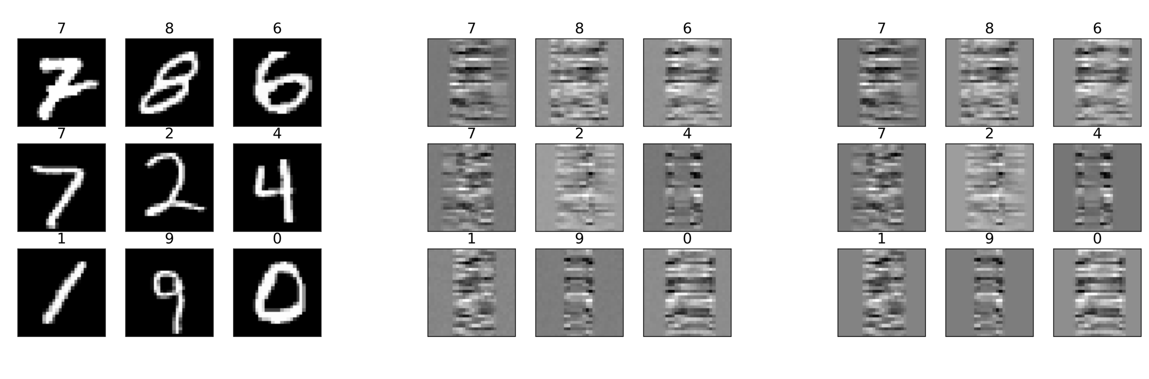
Figure 4 Comparisons of U_e from simulations and surrogates sampled at 10 different sensor locations.

Inverse Problem - Φ_θ

Recovering signals from sensors to the crack that might occur in a structure, is an inverse problem formulated by

$$y = \mathcal{A}(x) + \epsilon$$
$$x^* = \arg\min_x ||\mathcal{A}(x) - y||_2. \quad (4)$$

X, Y represent model parameter and measurement space, respectively. Furthermore, $\mathcal{A} : X \to Y$ is a continuous forward operator and ϵ is a sample noise during observation. This problem is ill-posed which means its solution is very sensitive to perturbations generated from signal collection.

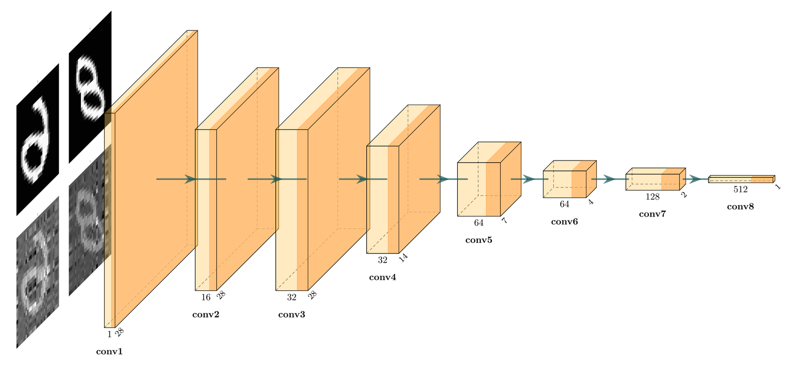


(a) ground truth **(b)** linear map **(c)** add noise

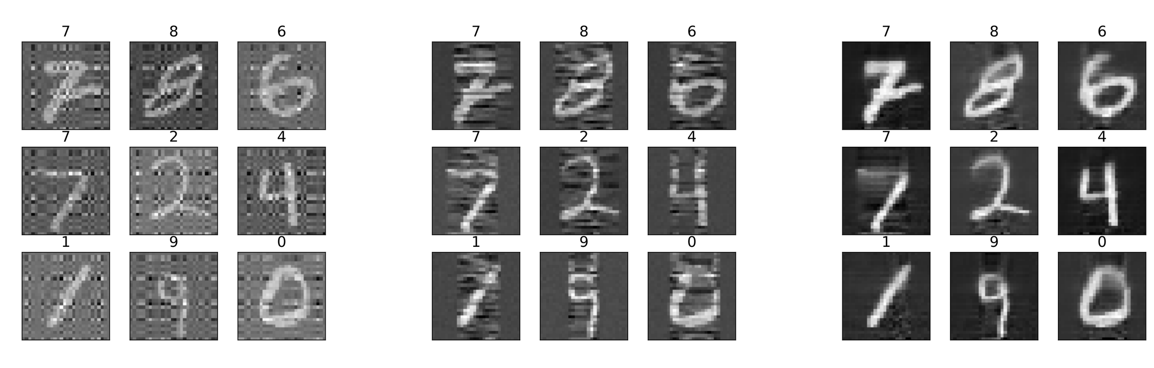
Figure 5 MNISIT dataset was used as an example to illustrate solution of inversion with the impact from noise. Specifically, a linear map was applied to images and added a Gaussian noise.

Adversarial Regularizer[2] (AdReg, $\Phi_\theta : X \mapsto \mathbb{R}$) shows promising ability to reduce the impact from perturbations

$$x^* = \arg\min_x \{||\mathcal{A}(x) - y||_2^2 + \Phi_\theta(x)\}, \quad (5)$$



The backbone of AdReg is a convoluntional Neural Networks (CNN). Ground truth and naive solution from the least square method are input into AdReg to learn the difference. The following experimental results demonstrated AdReg is able to give a good solution of inverse problem and could merge FNO together into full wave inversion.



(a) lstsq **(b)** L2 **(c)** AdReg

Figure 6 The figure contains some examples from MNIST and the solution of inverse problem by different methods. Adreg images are obtained from adversarial regularizers; L2 is Tikhonov regularization.

[1] Z. Li, N. Kovachki, K. Azizzadenesheli, B. Liu, K. Bhattacharya, A. Stuart, and A. Anandkumar, Fourier neural operator for parametric partial differential equations, 2020
[2] S. Lunz, O., "Oktem, and C.-B. Schönlieb, Adversarial regularizers in inverse problems, 2018

The authors gratefully acknowledge the support through the Georg Nemetschek Institut (GNI, project DeepMonitor), and by the Deutsche Forschungsgemeinschaft, project no. 468830823.